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PARAMETRIC EXCITATION OF A SECONDARY FLOW IN A VERTICAL LAYER OF A FLUID IN THE PRESENCE OF SMALL SOLID PARTICLES

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Thermal convection is studied in an inhomogeneous medium consisting of a fluid and a solid admixture under conditions of finite-frequency vibrations. Convection equations are derived within the framework of the generalized Boussinesq approximation, and the problem of flow stability in a vertical layer of a viscous fluid with horizontal oscillations along the layer to infinitely small perturbations is considered. A comparison with experimental data is made.

Introduction. The possibility of appearance of convective instability in a system with a periodically varying parameter was first noted by Gershuni and Zhukhovitskii [1] who considered a plane horizontal layer of a fluid heated from below; a periodic action on the layer was performed by modulation of the gravity force. This fact was experimentally confirmed in [2, 3]. It may be assumed that the plane-parallel flow in a vertical layer heated from the side, whose stability has been extensively studied for the last decades (see, for example, [4]), is also sensitive to a periodic external action. There are practically no theoretical and experimental papers dealing with this problem. Only Zyuzgin and Putin [3], who studied experimentally the effect of horizontal vibrations on the stability of an ascending-descending flow in a vertical layer of kerosene ($\mathrm{Pr} = 26$), found a secondary flow, which was not predicted theoretically in the high-frequency approximation. It was also noted in [3] that this flow appears if the vibration frequency is lower than 19 Hz, and the flow structure was described (vertical banks arising on the background of an ascending-descending flow).

At the same time, it was shown in a number of papers (see, for example, [5–7]) that a fine heavy admixture may exert, under certain conditions, a significant effect on flow stability and structure. The objective of the present work is to obtain model equations for convection in a dusty medium under conditions of finite-frequency vibrations, to study the effect of a solid admixture on flow stability, and to explain the new convection mode observed in the experiment.

1. Formulation of the Problem. Derivation of the Governing Equations. We consider a viscous incompressible fluid flow in an infinite vertical layer of thickness $2h$ with a constant temperature difference 2Θ at the boundaries. Let the origin be in the center of the layer, the x axis be directed across the layer, and the y and z axes be located in the plane of the layer vertically and horizontally, respectively. The fluid layer experiences the action of harmonic horizontal vibrations with an amplitude a and frequency ω in the plane of the layer, which are directed along the unit vector $\boldsymbol{n}$.

We also assume that the fluid contains a finely dispersed solid admixture. Let the quantities μ and φ characterize the fraction of the fluid and solid phases in a unit volume of the heterogeneous mixture: $\mu + \varphi = 1$. For each component of the medium, we write the balance equations for mass, momentum, and energy in the differential form:

$$\frac{\partial \mu\rho_{\mathrm{f}}}{\partial t} + \nabla\cdot(\mu\rho_{\mathrm{f}}\boldsymbol{v}_{\mathrm{f}}) = 0, \qquad \frac{\partial \varphi\rho_{\mathrm{s}}}{\partial t} + \nabla\cdot(\varphi\rho_{\mathrm{s}}\boldsymbol{v}_{\mathrm{s}}) = 0,$$

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$$\mu\rho_{\rm f}\Big(\frac{\partial \boldsymbol{v}_{\rm f}}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla\boldsymbol{v}_{\rm f}\Big)=-\mu\nabla p+\eta\nabla\cdot(\mu\varepsilon)+\varphi\alpha(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\mu\rho_{\rm f}g(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos{(\omega t)}),$$

$$\varphi\rho_{\rm s}\Big(\frac{\partial \boldsymbol{v}_{\rm s}}{\partial t}+\boldsymbol{v}_{\rm s}\cdot\nabla\boldsymbol{v}_{\rm s}\Big)=-\varphi\nabla p-\varphi\alpha(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\varphi\rho_{\rm s}g(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos{(\omega t)}), \tag{1}$$

$$\mu\rho_{\rm f}c_{\rm f}\Big(\frac{\partial T_{\rm f}}{\partial t}+\boldsymbol{v}_{\rm f}\cdot\nabla T_{\rm f}\Big)=æ\nabla\cdot(\mu\nabla T_{\rm f})+\varphi\zeta(T_{\rm s}-T_{\rm f}),\quad \varphi\rho_{\rm s}c_{\rm s}\Big(\frac{\partial T_{\rm s}}{\partial t}+\boldsymbol{v}_{\rm s}\cdot\nabla T_{\rm s}\Big)=-\varphi\zeta(T_{\rm s}-T_{\rm f}).$$

Here $\boldsymbol{v}_{\rm f}$ and $\boldsymbol{v}_{\rm s}$ are the velocities of the components of the mixture (hereinafter, the quantities denoted by the subscripts “f” and “s” refer to the fluid and solid phases, respectively), $T_{\rm f}$ and $T_{\rm s}$ are the temperatures of the fluid and solid phases, $\rho_{\rm f}$, $\rho_{\rm s}$ and $c_{\rm f}$, $c_{\rm s}$ are the densities and heat capacities of the fluid and solid phases, respectively, $æ$ is the thermal conductivity of the fluid, α and ζ are the coefficients of friction and heat transfer between the phases, $\boldsymbol{\gamma}$ is the unit vector directed along the y axis, ε is the tensor of viscous stresses, and $A=a\omega^2/g$ is the overloading parameter. We assume that the interaction between the phases obeys the Stokes law, and the heat transfer follows the Fourier law:

$$\alpha=6\pi r\eta/V,\qquad \zeta=4\pi ræ/(Vc_{\rm f}).$$

Here r and V are the characteristic size and volume of the particle, respectively.

Convective flows are usually studied on the basis of the Boussinesq approximation, where deviations of the medium density from the mean value are assumed to be so small that they may be ignored everywhere, except for the term with the lift force in the equation of motion. We used the simplest equation of state $\rho=\rho_0(1-\beta T)$, where β is the coefficient of volume expansion of the fluid. The equations in the Boussinesq approximation may be formally obtained as a result of the limiting transition for the parameter of temperature inhomogeneity $\beta\Theta$ tending to zero and the Galileo number ${\rm Ga}=gh^3/\nu^2$ tending to infinity. Their product $\beta\Theta\,{\rm Ga}={\rm Gr}$ (Grashof number) remains finite.

Following the approach described in detail in [5] for the steady case, we obtain the generalized Boussinesq equations assuming the characteristic size of the particles to be small as compared to the characteristic size of the cavity. Thus, the problem acquires one more asymptotic parameter r/h. We represent the fields of density, pressure, and temperature in the form of the sum of constant mean values $\rho_{\rm f0}$, p_0, $T_{\rm f0}$, and $T_{\rm s0}$ and small deviations from them $\rho_{\rm f}$, p, $T_{\rm f}$, and $T_{\rm s}$; the changes in the density of the solid admixture due to thermal expansion of the material is ignored. Collecting terms of the same order in Eq. (1), we obtain the following equations in the zero approximation:

$$\frac{\partial\mu}{\partial t}+\nabla\cdot(\mu\boldsymbol{v}_{\rm f})=0,\qquad \frac{\partial\varphi}{\partial t}+\nabla\cdot(\varphi\boldsymbol{v}_{\rm s})=0,$$

$$0=-\mu\nabla p_0+\varphi\alpha(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\rho_{\rm f0}\mu g(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos{(\omega t)}),$$

$$0=-\varphi\nabla p_0-\varphi\alpha(\boldsymbol{v}_{\rm s}-\boldsymbol{v}_{\rm f})-\rho_{\rm s0}\varphi g(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos{(\omega t)}), \tag{2}$$

$$0=\varphi\zeta(T_{\rm s}-T_{\rm f}),\qquad 0=-\varphi\zeta(T_{\rm s}-T_{\rm f}).$$

Eliminating the mean pressure from system (2), we obtain the relation between the velocities of the solid particles and fluid

$$\boldsymbol{v}_{\rm s}=\boldsymbol{v}_{\rm f}-\frac{\mu g}{\alpha}\,(\rho_{\rm s0}-\rho_{\rm f0})(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos{(\omega t)}), \tag{3}$$

which allows us to pass to the one-fluid model in the next order of expansion. We confine ourselves to consideration of the following limiting case: the particles are assumed to be heavy ($D\to\infty$), and their volume concentration is assumed to be small ($\varphi\to 0$); thus, the mass concentration of the admixture $\xi=\varphi D$ remains finite. We normalize the length to h, the time to h^2/ν, the velocity to ν/h, the temperature to Θ, and the pressure to $\rho_{\rm f0}\nu^2/h^2$. Then the dimensionless equations of convection take the following form:

$$(1+\xi)\Big(\frac{\partial \boldsymbol{v}_{\rm f}}{\partial t}+(\boldsymbol{v}_{\rm f}\cdot\nabla)\boldsymbol{v}_{\rm f}\Big)$$
$$=-\nabla p+\Delta\boldsymbol{v}_{\rm f}+\mathrm{Gr}\,T_{\rm f}(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos(\Omega t))+S\xi(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos(\Omega t))\cdot\nabla\boldsymbol{v}_{\rm f}-S\xi A\Omega\boldsymbol{n}\sin(\Omega t),$$
$$(1+B\xi)\Big(\frac{\partial T_{\rm f}}{\partial t}+(\boldsymbol{v}_{\rm f}\cdot\nabla)T_{\rm f}\Big)=\frac{1}{\mathrm{Pr}}\,\Delta T_{\rm f}+SB\xi(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos(\Omega t))\cdot\nabla T_{\rm f}, \tag{4}$$
$$\frac{\partial\xi}{\partial t}+(\boldsymbol{v}_{\rm f}\cdot\nabla)\xi=S(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos(\Omega t))\cdot\nabla\xi,\qquad \nabla\cdot\boldsymbol{v}_{\rm f}=0,$$
$$\boldsymbol{v}_{\rm s}=\boldsymbol{v}_{\rm f}-S(\boldsymbol{\gamma}+\boldsymbol{n}\,A\cos(\Omega t)),\qquad T_{\rm s}=T_{\rm f}.$$

The problem has seven dimensionless parameters: $\mathrm{Gr}=g\beta\Theta h^3/\nu^2$, $\mathrm{Pr}=\nu/\chi$ are the Grashof and Prandtl numbers, $A=a\omega^2/g$ and $\Omega=\omega h^2/\nu$ are the amplitude and frequency of vibrations, $S=\delta(D-1)\,\mathrm{Ga}\,r^2/h^2$ is the parameter of the two-phase medium, and $D=\rho_{\rm s0}/\rho_{\rm f0}$ and $B=c_{\rm s}/c_{\rm f}$ are the ratios of phase densities and heat capacities, respectively. We note that none of them coincides with the asymptotically large or small parameters Ga, $\beta\Theta$, D, or r/h, which indicates the correctness of the approach. This differs the model proposed here from the model proposed, for instance, in [8], which is not the Boussinesq model in this sense.

2. Main Flow. We confine ourselves to considering a situation, where the admixture concentration is independent of the coordinates: $\xi=\xi_0$. In this case, system (4) may be significantly simplified. In the equation of motion, the term proportional to $\sin(\Omega t)$ becomes potential, and it may be eliminated from consideration by pressure renormalization. Taking into account the boundary conditions

$$\boldsymbol{v}_{\rm f}=0,\qquad T_{\rm f}=\mp1\quad\text{for}\quad x=\pm1, \tag{5}$$

we find the main state of the system established for rather low values of the parameters Gr, A, and Ω. Assuming the flow to be plane–parallel, we seek the solution in the form $\boldsymbol{v}_{\rm f0}=\boldsymbol{V}(0,V_y(x),V_z(x,t))$ and $T_{\rm f}=T_0(x)$. Then, the solution for the y-component of velocity and temperature is

$$V_y=(1/6)\,\mathrm{Gr}\,(x^3-x),\qquad T_0=-x, \tag{6}$$

and the z-component of the main flow has a more complicated form:

$$V_z=\mathrm{Re}\Big(i\,\frac{\mathrm{Gr}\,A}{\Omega(1+\xi_0)}\Big(x-\sinh\sqrt{\frac{(1+\xi_0)\Omega}{2}}\,(1+i)x\Big/\Big(\sinh\sqrt{\frac{(1+\xi_0)\Omega}{2}}\,(1+i)\Big)\Big)\exp(i\Omega t)\Big). \tag{7}$$

A solution of the form of (7) was first obtained in [9], where such a flow oscillating with a finite frequency arose in a plane layer under conditions of zero gravity. We note, however, that the flow with a cubic velocity profile (6), which arises n a vertical layer due to the force of gravity, exerts a significant effect on the vibrational component of the flow, which is evidenced by the presence of the Grashof number in Eq. (7).

It follows from (7) that the component V_z makes a significant contribution to the main flow only for rather low vibration frequencies: $\Omega\ll1$. In this case, relation (7) almost coincides with Eq. (6) for the y-component of the main-flow velocity and may be represented in the form $V_z\approx(1/6)\,\mathrm{Gr}\,A(x^3-x)\cos(\Omega t)$. The right part of this relation describes a slow periodic change in the cubic profile of velocity. For $\Omega\to\infty$, we have $V_z\to0$. Thus, the governing parameter of the problem is actually the complex $A_g=\mathrm{Gr}\,A$.

3. Spectral–Amplitude Problem. We consider the stability of the main flow (6), (7) to infinitely small perturbations. Based on the data of [3], we ignore the dependence of all fields that describe the critical perturbations on the vertical coordinate ($\partial/\partial y=0$). We obtain

$$(1+\xi_0)\Big(\frac{\partial v_{{\rm f},y}}{\partial t}+V_z\,\frac{\partial v_{{\rm f},y}}{\partial z}+v_{{\rm f},x}\,\frac{\partial V_y}{\partial x}\Big)=\Delta_{xz}v_{{\rm f},y}+\mathrm{Gr}\,T+AS\xi_0\cos(\Omega t)\,\frac{\partial v_{{\rm f},y}}{\partial z}; \tag{8}$$

$$(1+\xi_0)\Big(\frac{\partial\boldsymbol{v}_{\rm f}}{\partial t}+V_z\,\frac{\partial\boldsymbol{v}_{\rm f}}{\partial z}+v_{{\rm f},x}\,\frac{\partial\boldsymbol{V}}{\partial x}\Big)=-\nabla p+\Delta_{xz}\boldsymbol{v}_{\rm f}+A_g\Big(T\boldsymbol{n}+\frac{S\xi_0}{\mathrm{Gr}}\,\frac{\partial\boldsymbol{v}_{\rm f}}{\partial z}\Big)\cos(\Omega t),$$

$$(1+B\xi_0)\Big(\frac{\partial T_{\rm f}}{\partial t}+V_z\,\frac{\partial T_{\rm f}}{\partial z}+v_{{\rm f},x}\,\frac{\partial T_0}{\partial x}\Big)=\frac{1}{\mathrm{Pr}}\,\Delta_{xz}T_{\rm f}+ASB\xi_0\cos(\Omega t)\,\frac{\partial T_{\rm f}}{\partial z}, \tag{9}$$

$$\frac{\partial v_{{\rm f},x}}{\partial x}+\frac{\partial v_{{\rm f},z}}{\partial z}=0,$$

where $\Delta_{xz} = \partial^2/\partial x^2 + \partial^2/\partial z^2$ and $\boldsymbol{v} = (v_{\mathrm{f},x}, v_{\mathrm{f},z})$. It follows from system (8), (9) that the problem of finding the y-component of the perturbation (8) may be solved independently if the functions $v_{\mathrm{f},x}$ and T_{f} are known. The remaining equations of (9) may be transformed as follows. We pass to a new frame of reference to eliminate the parameter S from the equation of motion and rescale the parameters and variables:

$$\bar{z} = z + \frac{S\xi_0 A}{\Omega(1+\xi_0)}\sin(\Omega t), \quad \bar{x} = x, \quad \bar{t} = \frac{t}{1+\xi_0}, \quad T = \frac{T_{\mathrm{f}}}{1+\xi_0},$$
$$\overline{\mathrm{Pr}} = \mathrm{Pr}\,\frac{1+B\xi_0}{1+\xi_0}, \quad \bar{S} = S\,\frac{\xi_0(B-1)A}{1+B\xi_0}, \quad \bar{A}_g = A_g(1+\xi_0), \quad \bar{\Omega} = \Omega(1+\xi_0). \tag{10}$$

Introducing the stream function, we obtain the following relations (the bar is left only over the parameters):

$$\frac{\partial \Delta\Psi}{\partial t} = \Delta^2\Psi - V_z\,\frac{\partial\Delta\Psi}{\partial z} + \frac{\partial^2 V_z}{\partial x^2}\,\frac{\partial\Psi}{\partial z} + \bar{A}_g\cos(\bar{\Omega}t)\,\frac{\partial T}{\partial x},$$
$$\frac{\partial T}{\partial t} = \frac{1}{\overline{\mathrm{Pr}}}\,\Delta T - V_z\,\frac{\partial T}{\partial z} - \frac{\partial\Psi}{\partial z} + \bar{S}\cos(\bar{\Omega}t)\,\frac{\partial T}{\partial z}. \tag{11}$$

Due to transformation (10), the number of dimensionless parameters determining the stability of the main flow reduced to four: $\bar{A}_g$, $\bar{S}$, $\overline{\mathrm{Pr}}$, and $\bar{\Omega}$. We consider only normal perturbations of the form $\Psi(x,z,t) = \varphi(x,t)\exp(ikz)$ and $T(x,z,t) = \theta(x,t)\exp(ikz)$, where k is the wavenumber along the z axis. As a result, we obtain the following spectral–amplitude problem for determining φ and θ:

$$\frac{\partial\Delta\varphi}{\partial t} = \Delta^2\varphi - ik\Big(V_z\Delta\varphi - \frac{\partial^2 V_z}{\partial x^2}\,\varphi\Big) + \bar{A}_g\,\frac{\partial\theta}{\partial x}\,\cos(\bar{\Omega}t),$$
$$\frac{\partial\theta}{\partial t} = \frac{1}{\overline{\mathrm{Pr}}}\,\Delta\theta - ik(V_z\theta + \varphi - \bar{S}\theta\cos(\bar{\Omega}t)); \tag{12}$$

$$\varphi = \frac{\partial\varphi}{\partial x} = 0, \quad \theta = 0 \quad \text{for} \quad x = \pm 1. \tag{13}$$

Here $\Delta = \partial^2/\partial x^2 - k^2$.

4. Numerical Solution. The spectral–amplitude problem (12), (13) was solved numerically using the Galerkin method. The sought amplitudes φ and θ were represented in the form of expansions with respect to the basis functions $\Phi_n(x)$ and $\Theta_n(x)$

$$\varphi(x,t) = \sum_{n=0}^{N/2} a_n(t)\Phi_n(x), \qquad \theta(x,t) = \sum_{n=0}^{N/2} b_n(t)\Theta_n(x), \tag{14}$$

which satisfy the following boundary conditions (13):

$$\Phi_n = \begin{cases} \dfrac{\cosh(kx)}{\cosh k} - \dfrac{\cos(\sqrt{\mu_n - k^2}\,x)}{\cos\sqrt{\mu_n - k^2}}, \\[2ex] \dfrac{\sinh(kx)}{\sinh k} - \dfrac{\sin(\sqrt{\mu_n - k^2}\,x)}{\sin\sqrt{\mu_n - k^2}}, \end{cases} \qquad \Theta_n = \begin{cases} \cos(\rho_n x), & n = 0, 2, 4, \ldots, \\ \sin(\rho_n x), & n = 1, 3, 5, \ldots\ . \end{cases} \tag{15}$$

Here μ_n and ρ_n are determined from the relations

$$\sqrt{\mu_n - k^2}\tan\sqrt{\mu_n - k^2} = k(-\tanh k), \quad n = 0, 2, 4, \ldots,$$
$$\sqrt{\mu_n - k^2}\cot\sqrt{\mu_n - k^2} = k\coth k, \qquad n = 1, 3, 5, \ldots, \qquad \rho_n = \frac{\pi}{2}\,(n+1).$$

Substituting expansions (14) into Eq. (12) and using the orthogonality conditions of the Galerkin method, we obtain the following system of N linear differential equations for the amplitudes $a_n(t)$ and $b_n(t)$:

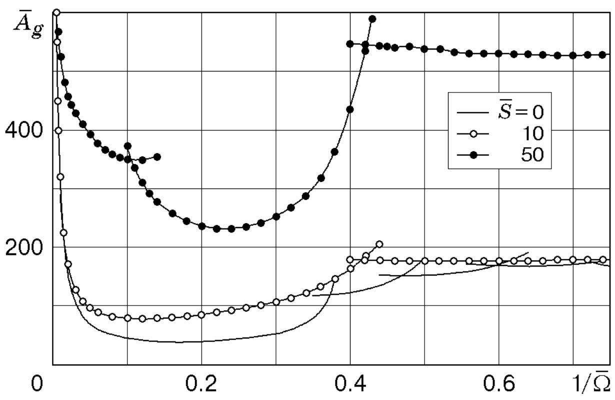

Fig. 1

$$\frac{da_n}{dt} = -\mu_n a_n - ik\sum_m A_{nm}a_m + \bar{A}_g\sum_m B_{nm}b_m\cos(\bar{\Omega}t), \qquad n = 0,\ldots,N/2,$$
$$\frac{db_n}{dt} = -\rho_n b_n - ik\sum_m C_{nm}b_m - ik\sum_m D_{nm}a_m + ik\bar{S}b_n\cos(\bar{\Omega}t). \tag{16}$$

Here the coefficients A_{nm}, B_{nm}, C_{nm}, and D_{nm} are periodic functions of time and depend on the parameters k, $\bar{A}_g$, and $\bar{\Omega}$.

To find the boundaries of stability of system (16), the Floquet method was used. The construction of the monodrome matrix, i.e., integration of system (16) for N linearly independent initial conditions on the section from $t = 0$ to $t = 2\pi/\bar{\Omega}$, was performed by the Runge–Kutta–Fehlberg of the fourth or fifth order. The multiplicators were calculated using an appropriate program from the standard IMSL library, which was based on the QR-algorithm. In the course of computations, those values of the governing parameters were found for which at least one multiplicator was on the unit circumference. This is the value corresponding to the neutral curve. To avoid the dependence of the results on the number of basis functions retained in expansions (14), special attention was paid to convergence of the results. In certain ranges of the parameters, the algorithm of solving the problem included an automatic increase in the number of the basis functions until the addition of new functions changed the computation result by less than 0.1%. Thus, the minimum number of modes N^* necessary to reach "saturation" was determined. Then the main computations were performed with this fixed number of modes N^*. For example, it was found that, for a homogeneous medium ($\bar{S} = 0$) to reach the "saturation" state, 15–20 modes are needed for high frequencies ($10 < \bar{\Omega} < 1000$) and 25–30 modes are needed for low frequencies ($\bar{\Omega} < 10$); for an inhomogeneous medium ($\bar{S} \neq 0$), these values were significantly greater: 30–35 modes for high frequencies and 60–70 modes for low frequencies. In the latter case, such a large number of basis functions retained in the expansion is related to oscillations of the intermediate results of computations near the limiting value.

All the computations were performed for a fixed Prandtl number $\overline{\mathrm{Pr}} = 26$, which corresponds to kerosene T-1 used as a working fluid [3]. Figure 1 shows the neutral curves that separate the region of stability and the region of parametric excitation of the secondary flow in the plane ($1/\bar{\Omega}$, $\bar{A}_g$) for different values of the parameter $\bar{S}$. It is seen that addition of a solid admixture to an oscillating flow makes the latter more stable to perturbations considered in the present paper.

It should be noted that all the most "dangerous" perturbations for $\bar{S} = 0$ refer to the "integer" type, i.e., the frequency of their oscillations coincides with the frequency of the external action. The same result was obtained in [9], which is close to the present paper in terms of the formulation of the problem. The fact of existence of only "integer" perturbations is apparently explained by the symmetry of the problem. Both system (11) itself and the main flow (7) are invariant to the following transformation:

$$S_{xz}(\Psi(x,z), T(x,z)) = (\Psi(-x,-z), -T(-x,-z)). \tag{17}$$

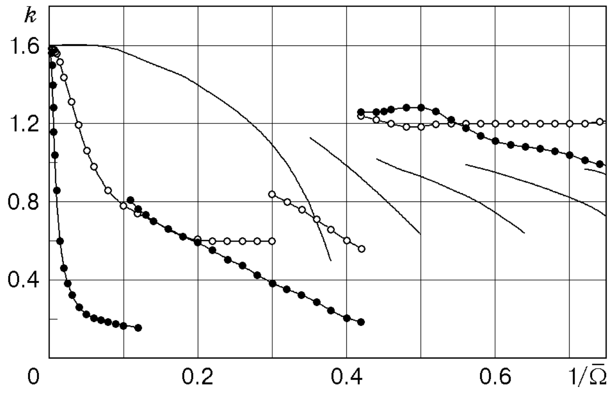

Fig. 2

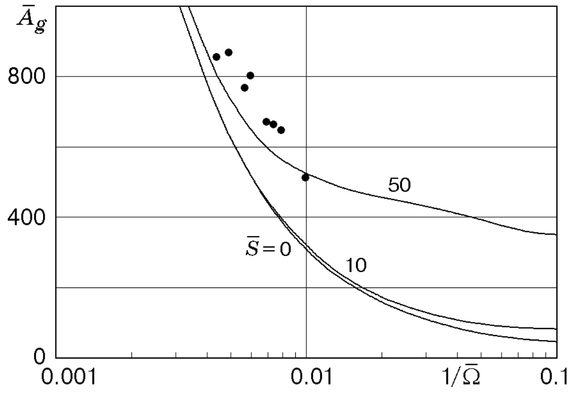

Fig. 3

Thus, within the phase space of the system, the main state is a cycle enclosed inside the invariant subspace determined by transformation (17). The branching of solutions may occur both inside the subspace and in the direction transverse to it. In the latter case, in situations of the general location, as is known from the bifurcation theory, there is always a branching pair of periodic solutions whose frequency is close to the frequency of oscillations of the initial cycle. The special feature of the system examined is that the branching of solutions inside the plane (at least, soft branching) is impossible. To show this, we write the dynamic system (16) in an extended form:

$$\frac{d}{dt}a_n^{\text{even}} = -\mu_n a_n^{\text{even}} - ik\sum_m A_{nm}a_m^{\text{odd}} + \bar{A}_g\sum_m B_{nm}b_m^{\text{odd}}\cos(\bar{\Omega}t),$$

$$\frac{d}{dt}a_n^{\text{odd}} = -\mu_n a_n^{\text{odd}} - ik\sum_m A_{nm}a_m^{\text{even}} + \bar{A}_g\sum_m B_{nm}b_m^{\text{even}}\cos(\bar{\Omega}t),$$

$$\frac{d}{dt}b_n^{\text{even}} = -\rho_n b_n^{\text{even}} - ik\sum_m C_{nm}b_m^{\text{odd}} - ik\sum_m D_{nm}a_m^{\text{even}},$$

$$\frac{d}{dt}b_n^{\text{odd}} = -\rho_n b_n^{\text{odd}} - ik\sum_m C_{nm}b_m^{\text{even}} - ik\sum_m D_{nm}a_m^{\text{odd}}.$$

The superscripts "even" and "odd" denote the amplitudes of even and odd modes, respectively. The transformation of symmetry (17) acquires the following form:

$$a_n^{\text{even}} \to (a_n^{\text{even}})^*, \quad a_n^{\text{odd}} \to -(a_n^{\text{odd}})^*, \quad b_n^{\text{even}} \to -(b_n^{\text{even}})^*, \quad b_n^{\text{odd}} \to (b_n^{\text{odd}})^*.$$

Here the asterisk denotes complex conjugation. To consider perturbations only inside the invariant plane, we assume the antisymmetric amplitudes to be equal to zero: $a_n^{\text{odd}} = b_n^{\text{even}} = 0$. We obtain

$$\frac{da_n^{\text{even}}}{dt} = -\mu_n a_n^{\text{even}} + \bar{A}_g\sum_m B_{nm}b_m^{\text{odd}}\cos(\bar{\Omega}t), \qquad \frac{db_n^{\text{odd}}}{dt} = -\rho_n b_n^{\text{odd}}.$$

It follows from here that all perturbations of this type decay. In the case of an inhomogeneous medium, the transformation of symmetry (17) is impossible. This means that all limitations on the bifurcation type disappear. In this case, indeed, both "semi-integer" and quasiperiodic solutions were found in the numerical study. The final answer to this question may be given only by studying the problem in the nonlinear formulation.

Figure 2 shows the minimum values of the perturbation wavenumber for different values of the parameter $\bar{S}$ (notation is the same as in Fig. 1). It follows from Fig. 2 that the solid admixture significantly decreases the wavenumber for high vibration frequencies and, vice versa, increases the wavenumber for low frequencies.

It should be noted that, though the critical perturbation becomes nonperiodic with addition of a solid admixture to the fluid, deviations from the "integer" type for high frequencies are so small that they are not of any practical importance.

5. Comparison with the Experiment. Figure 3 shows the calculation results obtained in the present work and the experimental data of [3]. The theoretical curve is constructed for the minimum values of the wavenumber. As is shown in Fig. 3, the calculation results are in good agreement with the experimental data (in the experiment, the instability of the main flow of the "integer" type was also registered above the neutral curve).

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